reported previously because of difficulties in measuring U_j at the required low speeds. However, spectral lines corresponding to an end mode were found for all jet speeds down to and including $U_j = 0$. Flow visualization experiments at $U_j = 0$ demonstrated the existence of a wake "bubble" at the end of the pipe similar to that found by Etzold and Fiedler (1976) for a solid cylinder.

V. Acknowledgment

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Remarks on Dusty Hypersonic Wedge Flow

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Introduction

THE study of the motion of solid particles in a dusty gas in the inviscid hypersonic shock layer of a slender wedge was initiated by Probstein and Fassio.[1] Their analysis was developed so as to determine the dust particle trajectories and the collection efficiency for the wedge. Peddieson and Lyu[2] extended their analysis to obtain closed-form solutions for the particle density and temperature distribution. In this Note we propose an alternate approach to the work of Ref. 2. In particular, we derive simpler formulas that allow an examination of the limiting behavior of the particle streamlines and density. We also give a simple physical interpretation related to the collection efficiency of the wedge and correct some results found in Ref. 2.

Analysis and Solution

We employ the notation of Ref. 2 and write the equations governing the dust cloud on a thin wedge in hypersonic flow as

$$(\rho_p u_p)_{,x} + (\rho_p v_p)_{,y} = 0 \tag{1a}$$

$$u_p u_{p,x} + v_p u_{p,y} = \alpha_1 \{(u-u_p)^2 + (v-v_p)^2\}^{(1-b)/2}(u-u_p) \tag{1b}$$

$$u_p v_{p,x} + v_p v_{p,y} = \alpha_1 \{(u-u_p)^2 + (v-v_p)^2\}^{(1-b)/2}(v-v_p) \tag{1c}$$

$$u_p T_{p,x} + v_p T_{p,y} = \alpha_2 (T - T_p) \tag{1d}$$

This formulation permits three interphase drag laws,[1] corresponding to low ($b=1$), intermediate ($b=3/5$), and high ($b=0$), Reynolds number ranges.

Equation (1a) implies the existence of a particle stream-function ψ_p defined by

$$\psi_{p,x} = -\rho_p v_p, \quad \psi_{p,y} = \rho_p u_p \tag{2}$$

Assuming that the clean gas is unaffected by the presence of the dust particles, we have

$$u=1, \quad v=0, \quad T=T_s \tag{3}$$

where T_s is the constant temperature of the gas in the shock layer. A von Mises transformation from coordinates (x,y) to (x,ψ_p) transforms the system of Eqs. (1) [using Eqs. (3)] to

$$(\rho_p u_p)_{,x} + \rho_p^2 (u_p v_{p,\psi_p} - v_p u_{p,\psi_p}) = 0 \tag{4a}$$

$$u_p u_{p,x} = \alpha_1 \{(1-u_p)^2 + v_p^2\}^{(1-b)/2}(1-u_p) \tag{4b}$$

$$u_p v_{p,x} = -\alpha_1 \{(1-u_p)^2 + v_p^2\}^{(1-b)/2} v_p \tag{4c}$$

$$u_p T_{p,x} = \alpha_2 (T_s - T_p) \tag{4d}$$

In addition, the equation for the particle paths is

$$\frac{dy}{dx} = \frac{v_p}{u_p} \tag{5}$$

Initial conditions associated with the system of Eqs. (4) and (5) are obtained from the shock relations as

$$u_p = 1, \quad v_p = -\theta_w, \quad T_p = 1, \quad y = x_s(\theta_s - \theta_w), \quad \rho_p = 1 \quad \text{at } x = x_s \tag{6}$$

where x_s is the point where the particle streamline along which the integration is being performed meets the shock wave.

Equations (4) and (5) can be solved subject to conditions (6) to obtain the variation of u_p, v_p, T_p, ρ_p, and y along the streamlines. For all values of b, we find

$$u_p = 1 \tag{7}$$

$$T_p = T_s - (T_s - 1)\exp[\alpha_2 (x_s - x)] \tag{8}$$

The solution for the other variables depend on b.

$b=1$:

$$v_p = -\theta_w \exp[\alpha_1 (x_s - x)] \tag{9a}$$

$$y = x_s(\theta_s - \theta_w) + (\theta_w/\alpha_1)\{\exp[\alpha_1 (x_s - x)] - 1\} \tag{9b}$$

$$\rho_p = [1 + (\theta_w/\theta_s)\{\exp[\alpha_1 (x_s - x)] - 1\}]^{-1} \tag{9c}$$

$b=3/5$:

$$v_p = -\theta_w [1 + (2\alpha_1/5)(-\theta_w)^{2/5}(x - x_s)]^{-5/2} \tag{10a}$$

$$y = x_s(\theta_s - \theta_w) - 5\theta_w^{3/5}/(3\alpha_1)\{1 - [1 + (2\alpha_1/5)(-\theta_w)^{2/5}(x - x_s)]^{-3/2}\} \tag{10b}$$

$$\rho_p = [1 + (\theta_w/\theta_s)\{[1 + (2\alpha_1/5)\theta_w^{2/5}(x - x_s)]^{-5/2} - 1\}]^{-1} \tag{10c}$$

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$b=0$:

$$v_p=[\alpha_1(x_s-x)-1/\theta_w]^{-1} \quad (11a)$$

$$y=x_s(\theta_s-\theta_w)-\alpha_1^{-1}\ln[1+\alpha_1\theta_w(x-x_s)] \quad (11b)$$

$$\rho_p=\{1-\theta_w/\theta_s+\theta_s^{-1}[\alpha_1(x-x_s)+1/\theta_w]^{-1}\}^{-1} \quad (11c)$$

Discussion

1) The results for u_p, v_p, and y are equivalent to those of Peddieson and Lyu.[2] The expression for T_p is also equivalent, but our result is in a much simpler form and does not depend explicitly on the drag law used (i.e., the value of b).

2) When $b=3/5$, our result for ρ_p is equivalent to the result of Peddieson and Lyu. However, when $b=1$, their result for ρ_p should read

$$\rho_p=[1-(\alpha_1/\theta_s)(z_s-z)]^{-1}$$

and when $b=0$, their result for ρ_p should be

$$\rho_p=[1-(\theta_w/\theta_s)\{1-\exp[\alpha_1(z-z_s)]\}]^{-1}$$

These corrected results are then equivalent to our results (9c) and (11c), respectively.

3) We note that for all values of b,

$$\rho_p\rightarrow\theta_s/(\theta_s-\theta_w)=1/\epsilon \text{ as } x\rightarrow\infty$$

where ϵ is the gas density ratio across the shock. The results of Peddieson and Lyu incorrectly indicate that $\rho_p\rightarrow\infty$ as $x\rightarrow\infty$.

4) For $b=0$, all particles strike the wedge, since $y>0$ for all x_s as $x\rightarrow\infty$. However, for $b=1$, particles will not strike the wedge if $x_s>\theta_w/[\alpha_1(\theta_s-\theta_w)]$, and for $b=3/5$, particles will not strike the wedge if $x_s>5\theta_w^{3/5}/[3\alpha_1(\theta_s-\theta_w)]$.

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Shape Factors between Coaxial Annular Disks Separated by a Solid Cylinder

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Nomenclature

x,y,z	= Cartesian coordinates of dA_1, cm
l_1,m_1,n_1	= cosines (i.e., direction cosines) of the angles between the normal to dA_1 and the x,y, and z axes, respectively
x_2,y_2,z_2	= Cartesian coordinates of a point on the periphery of surface 2, cm
S	= distance between dA_1 and a point on the periphery of surface 2, cm
r	= radial coordinate in plane of surface 2, cm
r_o,r_i	= outer and inner radii, respectively, of surface 2, cm
r_c	= radius of cylinder, cm
ρ	= radial coordinate in plane of dA_1, cm
θ	= angular displacement from x axis, rad
ϵ	= emissivity
ϕ,ω	= viewing angles, rad
h	= vertical distance between surface 1 and surface 2, cm

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Introduction

THE calculation of shape factors between parallel, coaxial annular disks with identical radii separated by a solid coaxial cylinder having the same radius as the inner radii of the disks is a relatively easy task which can be performed with the aid of shape-factor algebra and the closed-form expressions given in Hamilton and Morgan.[1] However, this is not the case when it is necessary to evaluate the shape factors between disks with different radii. This latter geometry, for which there is no closed-form expression available in the literature, is oftentimes encountered in thermal control calculations for spin-stabilized spacecraft and in discrete-element thermal models for annular fin-tube radiators for spacecraft applications.

In their analysis of annular fin-tube radiators, Sparrow et al.[2] used the contour integration method[3] to evaluate the radial dependence of shape factors between differential areas on opposing fins. However, this analysis, which required the numerical solution of a nonlinear, integro-differential equation for the temperature distribution in a fin, was carried out only for black ($\epsilon=1$) surfaces and no incident radiation from external sources (i.e., direct solar, albedo, and Earth IR). Since any realistic, spacecraft radiator design must account for nonblack ($\epsilon<1$) surfaces and external radiant heating, the most convenient method for analyzing such a system requires the formulation of a discrete-element thermal model in which each fin is divided up into a number of finite annular segments. Thus, it is necessary to evaluate the shape factors between the annular segments of the opposing fins.

The contour integral method is used in the present analysis to derive a closed-form expression for the shape factor from a differential area on one fin to a finite-sized annular area on the opposing fin. This expression can then be integrated numerically to obtain the desired shape factors between finite-sized annular areas on opposing fins. Results are given for some typical fin-tube geometries.

Analysis

Consider the geometry shown in Figs. 1 and 2 which illustrate the nomenclature used in this presentation. Throughout this discussion, surface 2 is always considered to be of finite size; surface 1 can be either a differential area or a finite-sized area, depending on the problem under consideration.

Sparrow[3] has shown that $F_{dA_1-A_2}$ can be expressed as the sum of three contour integrals in the following manner:

$$F_{dA_1-A_2}=l_1\oint_C\frac{(z_2-z)dy_2-(y_2-y)dz_2}{2\pi S^2}+m_1\oint_C\frac{(x_2-x)dz_2-(z_2-z)dx_2}{2\pi S^2}+n_1\oint_C\frac{(y_2-y)dx_2-(x_2-x)dy_2}{2\pi S^2} \quad (1)$$